Laminar Mixing of Heterogeneous Axisymmetric Coaxial Confined Jets

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Confined laminar mixing of dissimilar circular axisymmetric jets is studied. The system considered is binary, isothermal, and nonreacting. The central jet consists of a slow-moving heavy gas and the coflowing annular stream is a fast-moving light gas. This system is characteristic of a gas-core nuclear reactor where minimum mixing is desired. The boundary-layer equations are used to describe the confined jet mixing problem and the solution is obtained numerically by an explicit finite difference scheme. Solutions were obtained for a wide range of velocity ratio, density ratio, radius ratio, viscosity ratio, and outer stream Reynolds number and Schmidt number. Results are obtained in the form of velocity and mass fraction fields, and radial profiles for some typical runs are presented. The results show that an increase in the jet velocity U_1 causes an increase of the mass fraction potential core length $L_{\omega 1}$, slower decay of the centerline velocity and mass fraction, and a wider jet as well as larger developing length. Further, an increase in the jet density ρ_1 reduces the centerline velocity in the mixing region and increases the developing length.

Nomenclature

D_{12} = diffusion coefficient for binary system
M_i = molecular weight of component i
m = transverse coordinate for discretized problem
n = axial coordinate for discretized problem
$N_{Re,2}$ = Reynolds Number of outer stream, $N_{Re,2} = 2(R - R_1)U_2/\nu_2$
$N_{Sc,2}$ = Schmidt Number based on outer stream, $N_{Sc,2} = \nu_2/D_{12}$
p = static pressure
R = radius of confining pipe
R_1 = radius of inner jet tube
r = radial coordinate
U = average axial velocity of over-all flow
U_1 = average axial velocity of inner stream
U_2 = average axial velocity of outer stream
v_r = mass average radial velocity
v_z = mass average axial velocity; nondimensional axial velocity
$v_{z,1}$ = nondimensional centerline axial velocity
x_i = mole fraction of species i
z = axial coordinate
Δz = step size in z direction
$\Delta \phi$ = step size in ϕ direction
η = containment factor,

$$\eta = 2\pi \int_0^z \int_0^R \rho\omega_1 r dr dz \Big/ \rho_{p,1}\pi R_1^2 z$$

μ = dynamic viscosity
ν = kinematic viscosity
ρ = mass average density
Φ = value of ϕ at confining pipe wall
ϕ = transverse coordinate in ϕ-z plane
ψ = Stokes' stream function; transverse coordinate in von Mises plane
ω_i = mass fraction of species i
$\omega_{1,w}$ = wall mass fraction

Subscripts

1 = refers to inner stream, i.e., species 1
2 = refers to outer stream, i.e., species 2
i = species i
p = pure component

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Introduction

ALTHOUGH laminar jet mixing has been investigated for both incompressible and compressible cases, most of the work is limited to the unconfined, i.e., freejet mixing. The more complex case of confined jet mixing has not been studied in sufficient detail. This type of mixing occurs in jet pumps, ejectors, jet engine combustion chambers and in coaxial gaseous core reactors for nuclear rockets,[1–3] where a low-velocity fissionable gas is ejected coaxially into a high-velocity hydrogen propellant stream. For optimum performance of such an engine, it is necessary to reduce the loss of the fissionable component, i.e., to minimize mixing. In order to study this complex flow, the investigation of the mixing of laminar, isothermal, nonreacting confined heterogeneous jets is useful.

In 1964, Wood[4] studied analytically, as well as experimentally, the mixing of confined jets. For nearly equal entrance velocities, the calculated concentration profiles compare well with the experimental data. In 1966, Seider[5] performed analytical studies of laminar incompressible homogeneous confined jet mixing with and without chemical reaction. For the nonreacting case, the results are in good agreement with the concentration measurements of Wood. Fejer et al.,[6] carried out experimental as well as analytical studies of the confined mixing of coaxial streams; and pointed out that the results are in good agreement with Wood's[4] data for wall concentration. Unsteady as well as steady homogeneous mixing of confined laminar jets were studied by Agarwal and Torda.[7] Agreement with Seider's results is satisfactory.

In the present investigation, laminar coaxial confined heterogeneous mixing of incompressible jets is being studied analytically. The objective is to obtain the velocity and concentration fields in the mixing and developing regions, and to determine effects of the various parameters on mixing in order to understand the basic flow phenomena occurring in confined coaxial jet mixing.

Mathematical Model

The jet mixing problem to be studied is represented mathematically by the boundary-layer equations with appropriate boundary conditions. Auxiliary expressions are used to determine the local thermodynamic and transport properties of the fluid medium.

The use of the boundary-layer equations may be supported by the success with which they have been applied in the investigation of unconfined mixing.[8,9] Their application to the confined jet problem is justified in Ref. 7 and also in the discussion of the results in this paper. Thus, the mathematical model is based on the following assumptions: 1) the boundary-layer assumptions; 2) steady-state, isothermal flow without body forces and chemical reaction; 3) incompressible component fluids; and 4) invariance of binary diffusivity D_{12} with concentration. A typical jet entrance region for confined jet mixing is shown in Fig. 1a.

Formulation of Problem in Physical Plane (r,z)

Continuity equation

$$(\partial/\partial r)(\rho r v_r) + (\partial/\partial z)(\rho r v_z) = 0 \tag{1}$$

Momentum equation

$$\rho v_r \frac{\partial v_z}{\partial r} + \rho v_z \frac{\partial v_z}{\partial z} = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r \frac{\partial v_z}{\partial r}\right) \tag{2}$$

Diffusion equation

$$\rho v_r \frac{\partial \omega_1}{\partial r} + \rho v_z \frac{\partial \omega_1}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho D_{12} \frac{\partial \omega_1}{\partial r}\right) \tag{3}$$

Expression for density[10]

$$\rho = \frac{\omega_1/M_1}{\omega_1/M_1 + \omega_2/M_2}[\rho_{p,1} - \rho_{p,2}] + \rho_{p,2} \tag{4}$$

Expression for viscosity of a binary mixture[10]

$$\mu = \sum_{i=1}^{2} x_i \mu_{p,i} \Big/ \sum_{j=1}^{2} x_j \phi_{ij} \tag{5}$$

where

$$\phi_{ij} = \frac{1}{(8)^{1/2}}\left[1 + \frac{M_i}{M_j}\right]^{-1/2}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{1/2}\left(\frac{M_j}{M_i}\right)^{1/4}\right]^2 \tag{6}$$

In the present analysis D_{12} is constant, but for generality it is retained as a variable. For a unique solution, the pressure gradient dp/dz must be either prescribed or computed for any flow problem governed by the boundary-layer equations. Here, dp/dz was determined from the conservation of mass flow rate across any cross section in the flow region. Use of this constraint together with the momentum equation yields an explicit equation for dp/dz. This equation will be referred to as the equation of constraint.

Equation of Constraint

$$\frac{dp}{dz} = \frac{1}{\int_0^R \frac{r}{v_z}dr}\int_0^R \frac{r}{v_z} \times \left[v_z^2 \frac{\partial\omega_1/\partial z}{M_1 M_2\left[\frac{\omega_1}{M_1} + \frac{\omega_2}{M_2}\right]^2}[\rho_{p,1} - \rho_{p,2}] - \rho v_r \frac{\partial v_z}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left[\mu r \frac{\partial v_z}{\partial r}\right]\right] dr \tag{7}$$

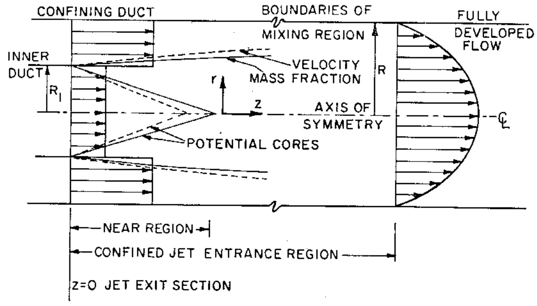

Fig. 1a Typical confined jet entrance region and coordinate system.

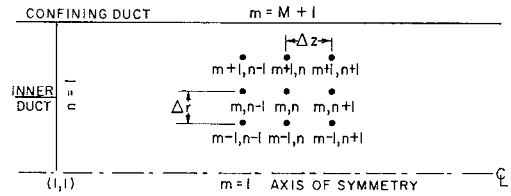

Fig. 1b Discretized rectangular grid system in jet entrance region.

Boundary Conditions

1) At the initial section, i.e., at $z = 0$

$$v_z(r,0) = \begin{cases}\lambda_1(r) & \text{if } 0 \le r \le R_1 \\ \lambda_2(r) & \text{if } R_1 < r \le R\end{cases}$$
$$v_r(r,0) = 0 \quad \text{if } 0 \le r \le R \tag{8}$$
$$\omega_1(r,0) = \begin{cases}\lambda_3(r) & \text{if } 0 \le r \le R_1 \\ \lambda_4(r) & \text{if } R_1 < r \le R\end{cases}$$

In this study $\lambda_1(r)$, $\lambda_2(r)$, $\lambda_3(r)$, and $\lambda_4(r)$ were chosen to be constants.

2) At the centerline, i.e., at $r = 0$

$$v_r(0,z) = (\partial v_z/\partial r)|_{r=0} = (\partial\omega_1/\partial r)|_{r=0} = 0 \tag{9}$$

3) At the wall, i.e., at $r = R$

$$v_z(R,z) = v_r(R,z) = (\partial\omega_1/\partial r)|_{r=R} = 0 \tag{10}$$

Equations (1–10) complete the formulation of the jet mixing problem. The von Mises transformation is used in this analysis for obtaining the solution.

von Mises Transformation

The stream function is defined by

$$\partial\psi/\partial r = \rho v_z r, \; \partial\psi/\partial z = -\rho v_r r \tag{11}$$

and the inverse transformation is given by

$$r^2 = 2\int_0^\psi \frac{d\psi}{\rho v_z} \tag{12}$$

In this study, the number of ψ grid points representing the inner stream decreases as the ratio of the mass fluxes of the outer to the inner streams increases. Hence, to obtain a proper finite difference representation of the inner stream for these large mass flux ratios without unreasonably increasing the number of steps in ψ direction, a transformation is used to stretch the ψ coordinate in the region of the inner stream. This transformation, denoted as the ϕ transformation, and the corresponding transformed equations and boundary conditions are presented next.

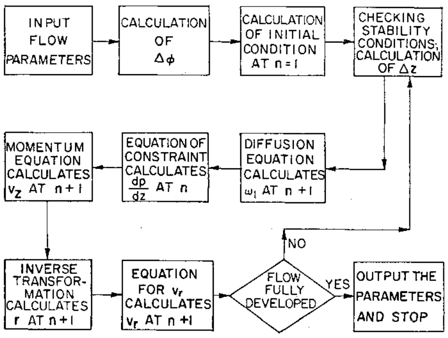

Fig. 2 Flow diagram for numerical solution.

Formulation of Problem in ϕ-Transform Plane (ϕ,z)

The ϕ transformation is defined by

$$\phi = \psi^{1/\alpha} \tag{13}$$

where

$$\alpha = \begin{cases} 1 & \text{when } \rho_1 U_1/\rho_2 U_2 \simeq 1 \\ 2 & \text{when } \rho_1 U_1/\rho_2 U_2 \ll 1 \end{cases}$$

The equations in the von Mises plane correspond to $\alpha = 1$. The inverse ϕ transformation is defined by

$$\int_{r_1}^{r_2} r dr = \int_{\phi_1}^{\phi_2} \frac{d\phi}{\rho v_z \, d\phi/d\psi} \tag{14}$$

Equations (2, 3, and 7) transform to

$$\frac{\partial v_z}{\partial z} = \frac{-1}{\rho v_z}\frac{dp}{dz} + \frac{d\phi}{d\psi}\frac{\partial}{\partial\phi}\left[\mu r^2 \rho v_z \frac{d\phi}{d\psi}\frac{\partial v_z}{\partial \phi}\right] \tag{15}$$

$$\frac{\partial \omega_1}{\partial z} = \frac{\partial\phi}{\partial\psi}\frac{\partial}{\partial\phi}\left[r^2\rho^2 D_{12} v_z \frac{d\phi}{d\psi}\frac{\partial\omega_1}{\partial\phi}\right] \tag{16}$$

$$\frac{dp}{dz} = \frac{1}{\displaystyle\int_0^\Phi \frac{d\phi}{\rho v_z^2 \, d\phi/d\psi}} \int_0^\Phi \times$$

$$\left[\begin{array}{l} -\rho v_r r \dfrac{d\phi}{d\psi}\dfrac{\partial}{\partial\phi}(\rho v_z) \\ + v_z \dfrac{\partial\omega_1}{\partial z}\left\{\dfrac{\rho_{p,1} - \rho_{p,2}}{M_1 M_2\left[\dfrac{\omega_1}{M_1} + \dfrac{\omega_2}{M_2}\right]^2}\right\} \\ + \rho \dfrac{d\phi}{d\psi}\dfrac{\partial}{\partial\phi}\left[\mu r^2 \rho v_z \dfrac{d\phi}{d\psi}\dfrac{\partial v_z}{\partial\phi}\right] \end{array}\right] \frac{d\phi}{\rho v_z \dfrac{d\phi}{d\psi}} \tag{17}$$

The continuity equation (1) is satisfied identically by the definition of the stream function. For variable density flows, however, the radial velocity v_r can be evaluated from Eq. (1) by solving it for $\partial(rv_r)/\partial r$. Thus, the radial velocity appearing in Eq. (17) is evaluated from the equation of continuity in the transformed plane

$$\frac{d\phi}{d\psi}\frac{\partial}{\partial\phi}(rv_r) = r\frac{v_r}{v_z}\frac{d\phi}{d\psi}\frac{\partial v_z}{\partial\phi} - \frac{1}{\rho v_z}\frac{\partial v_z}{\partial z} - \frac{1}{\rho^2}\frac{\partial\rho}{\partial z} \tag{18}$$

The auxiliary expressions (4–6) for ρ, μ remain unaffected by the transformation.

Boundary Conditions

The transformed boundary conditions are

1) At the initial section, i.e., $z = 0$,
v_z, v_r and ω_1 are specified functions of ϕ (19)

2) At the centerline, i.e., at $\phi = 0$

$$v_r(0,z) = (\partial v_z/\partial\phi)|_{\phi=0} = (\partial\omega_1/\partial\phi)|_{\phi=0} = 0 \tag{20}$$

3) At the wall, i.e., at $\phi = \Phi$

$$v_z(\Phi,z) = v_r(\Phi,z) = (\partial\omega_1/\partial\phi)|_{\phi=\Phi} = 0 \tag{21}$$

On the centerline, Eqs. (15) and (16) become

$$\partial v_z/\partial z = -(1/\rho v_z)dp/dz + \mu\partial^2 v_z/\partial\phi^2 \tag{22}$$

$$\partial\omega_1/\partial z = \rho D_{12}\partial^2\omega_1/\partial\phi^2 \tag{23}$$

Thus, the flow problem is completely represented by Eqs. (15–18) [together with Eqs. (22) and (23) for the centerline] and the boundary conditions (19–21).

Method of Solution

A forward-marching all-explicit numerical method is used in this analysis. The discretized rectangular grid and the coordinate system used to solve the problem are shown in Fig. 1b. For the transverse derivatives, central differences are used in the interior of the duct and backward differences are used at the duct wall. Forward differences are used for axial derivatives everywhere except in the equation for determining v_r where the axial derivatives are approximated by backward differences; the reason for this is given in Ref. 7.

The stability conditions for the finite difference equations are obtained by using the criterion developed by Karplus.[11] These conditions are realizable for nonnegative axial velocities, and are similar to those obtained by von Neumann's method.[12]

Stability Conditions

For momentum equation (15), $\Delta\phi$ is not limited from stability considerations and is selected from the required resolution and the accuracy of the flow problem, and

$$\Delta z < [(1/2\nu)(1/r^2\rho^2 v_z)\{1/(d\phi/d\psi)^2\}]_{m,n}\Delta\phi^2 \tag{24}$$

For the diffusion equation (16), there is no restriction on $\Delta\phi$, and

$$\Delta z < [(N_{Sc}/2\nu)(1/r^2\rho^2 v_z)\{1/(d\phi/d\psi)^2\}]_{m,n}\Delta\phi^2 \tag{25}$$

The stability conditions of Eqs. (22) and (23) are less stringent than conditions (24) and (25). The more restrictive of conditions (24) and (25) is utilized in the numerical solution. Equations (17), (18), and the auxiliary expressions (4–6) are unconditionally stable.

The sequence of operations for obtaining the numerical solution is summarized in the simplified flow diagram presented in Fig. 2. An IBM 360/40 computer is used to solve the flow equations and the time required to obtain the solution for a typical case is approximately 10 min.

Results and Discussion

A detailed parametric study of confined jet mixing using laminar boundary-layer equations was carried out. The validity of the boundary-layer equations for the present flow conditions was checked by calculating second-order derivatives of the axial velocity v_z and the mass fraction ω_1. The axial derivatives were at least three orders of magnitude smaller than the corresponding transverse derivatives. Also, the radial velocity profiles showed only a low net radial flow, so that the normal pressure gradient could indeed be ne-

Table 1 Range of parameters

U_2/U_1	ρ_1/ρ_2	R_1/R	$N_{Re,2}$	$N_{Sc,2}$	ρ_2/ρ_1
1–30	1–8.3	0.28–0.563	1000–2000	0.75–2.0	0.75–2.0

glected, thus indicating that the boundary-layer assumptions hold good for confined jet mixing even in the near region for the range of parameters considered. Similar checks were used by Weinstein and Todd[9] to justify the use of the boundary-layer equations for unconfined mixing of streams with velocity ratio $U_2/U_1 = 100$ and density ratio $\rho_1/\rho_2 = 100$.

A few remarks regarding the use of the laminar flow model in the present analysis may also be in order. Very limited experimental results concerning the stability of confined jet mixing flows are available at the present. Ragsdale et al.[13] and Taylor and Masser[14] carried out experiments with a bromine-air system having a radius ratio $R_1/R = 0.125$ and predicted the range of laminar flow for such a jet mixing system. From these experiments, it is felt that the range of parameters considered in the present study is approximately within the range of laminar flow. Moreover, consideration of a wide range of parameters better enables the study of the over-all trends in the flow.

Direct comparison of the results with experimental data is not possible. Therefore, the problem of laminar confined mixing of jets of equimolecular weight streams was solved and the concentration profiles were compared with the experimental results of Wood.[4] Figure 3 shows that good agreement is obtained. Finally, as a partial check on the results of the present problem, the fully developed values of the flow parameters $v_z(r,z)$, $\omega_1(r,z)$, and dp/dz were compared with the corresponding asymptotic values that were obtained independently from theoretical considerations. The agreement of these asymptotic values gave further reliance on the numerical results.

A total of 57 cases was investigated. The basis of these runs was an air-Freon system. The range of values of the parameters studied includes many physical systems of practical interest. This range is shown in Table 1.

The transformation of the problem from the physical plane to the von Mises plane, or further to the ϕ-z plane, was performed mainly to avoid numerical instabilities at high-velocity ratio U_2/U_1. However, it was found later that when the velocity ratio U_2/U_1 reached a value where a stable solution was not obtainable, increasing the density ratio ρ_1/ρ_2 stabilized the solution. Therefore, it appears that the mass flux ratio $U_2\rho_2/U_1\rho_1$, rather than the velocity ratio U_2/U_1 alone, is the governing parameter for a stable convergent numerical solution. Similarly, decreasing the radius ratio R_1/R further increases the range of mass flux ratios for which a stable solution can be obtained. It was observed that the other parameters of the problem also affected the range for which stable solutions are obtainable. An attempt was made to obtain a correlation of all the parameters showing the bounds of the stable region, but no suitable correlation was found. Stable solutions were not obtainable for values of parameters beyond the range for which results are presented.

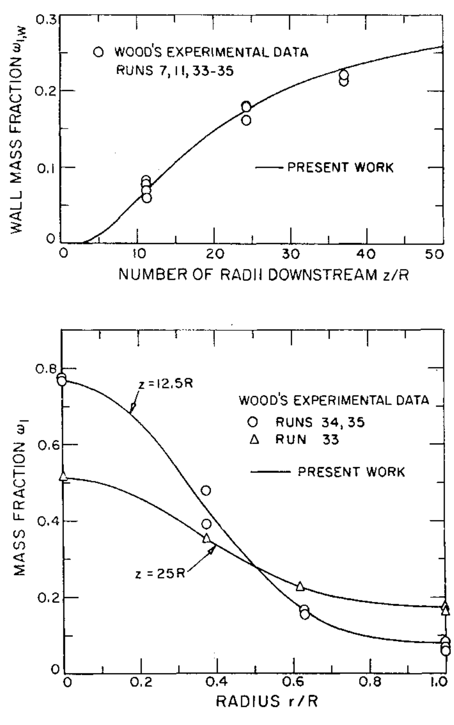

Fig. 3 Comparison of mass fraction profiles with Wood's experimental data.

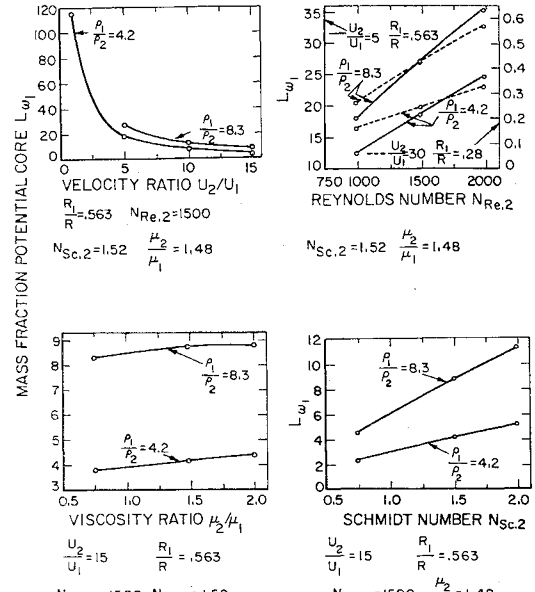

Fig. 4 Effects of variations of parameters on mass fraction potential core $L_{\omega 1}$.

Because of the dominating nonlinear effects, some of the investigated cases demanded a step size considerably smaller than those predicted by stability analysis. These cases have been studied only for small distances downstream because of the increased computer time requirements. In the graphical results presented the velocity has been made nondimensional with respect to U_1.

The results of the 57 cases investigated for the parametric study present the effects of six parameters—U_2/U_1, ρ_1/ρ_2, R_1/R, $N_{Re,2}$, $N_{Sc,2}$, and μ_2/μ_1—on the mass fraction potential core L_{ω_1}, the centerline velocity $v_{z,1}$, the wall mass fraction $\omega_{1,w}$, and the containment factor η. The variation of these six parameters was achieved by varying U_1, ρ_1, R_1, U_2, D_{12}, and μ_1, respectively. Larger $L_{\omega 1}$ and η, and smaller $v_{z,1}$ are desired for minimum mixing, whereas, to reduce spreading of the inner jet, smaller $\omega_{1,w}$ is essential.

Effects of Flow Parameters on $L_{\omega 1}$§

Figure 4 presents the effects of the various parameters on the length of the mass fraction potential core $L_{\omega 1}$. As U_2/U_1 decreases or ρ_1/ρ_2 increases, the momentum deficiency between the two streams is reduced, thereby resulting in slower mixing and, consequently, larger $L_{\omega 1}$. Also, retarded diffusion, arising from higher $N_{Sc,2}$ or relatively smaller viscous effect owing to increase in $N_{Re,2}$, yield a larger $L_{\omega 1}$. The effect of change of $N_{Re,2}$ on $L_{\omega 1}$ remain similar when R_1/R is reduced and U_2/U_1 is simultaneously increased substantially.

§ $L_{\omega 1}$ is the value of z where the mass fraction ω_1 at the centerline has changed by less than 5 % from its original centerline value.

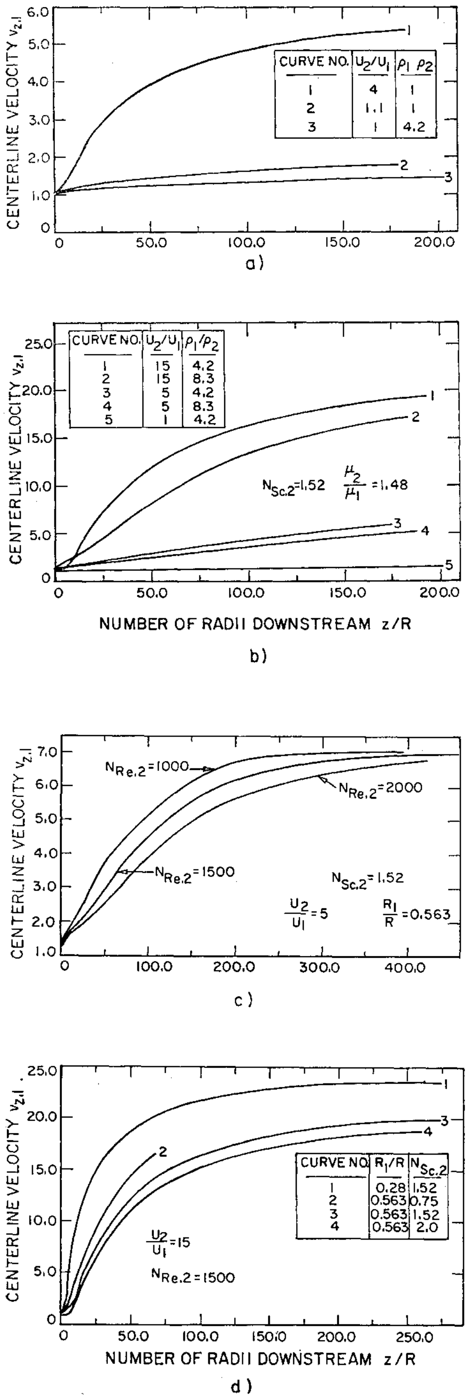

Fig. 5 Centerline velocity vs downstream distance: a) and b) $R_1/R = 0.563$, $N_{Re,2} = 1500$; c) and d) $\rho_1/\rho_2 = 4.2$, $\mu_2/\mu_1 = 1.48$.

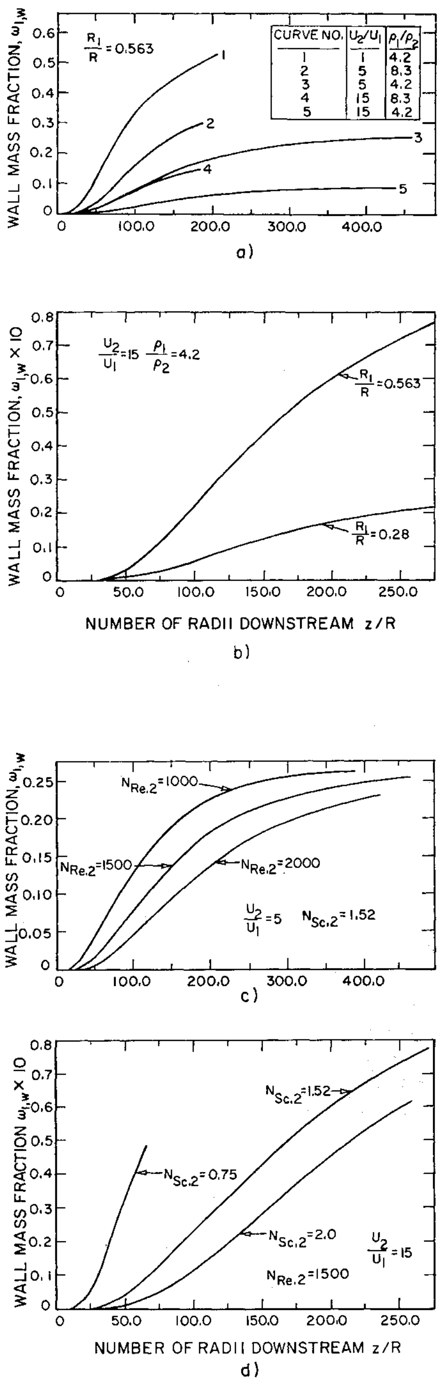

Fig. 6 Wall mass fraction vs downstream distance: a) and b) $N_{Re,2} = 1500$, $N_{Sc,2} = 1.52$, $\mu_2/\mu_1 = 1.48$; c) and d) $\rho_1/\rho_2 = 4.2$, $R_1/R = 0.563$, $\mu_2/\mu_1 = 1.48$.

Effects of Flow Parameters on $v_{z,1}$

Figure 5 presents the effects of the parameters on the centerline axial velocity $v_{z,1}$. For incompressible flow, the asymptotic centerline velocity depends only on the radius ratio R_1/R and the entrance velocities, so that the effect of the other parameters is confined to the mixing region only. An increase in ρ_1/ρ_2 reduces the velocity in the mixing region, since comparatively heavier inner jet has now to be accelerated, while an increase in R_1/R increases the amount of heavier fluid, thereby retarding the development of the flow. Also, the centerline velocity in the mixing region is lowered for higher $N_{Re,2}$ and $N_{Sc,2}$.

Effects of Flow Parameters on $\omega_{1,w}$

The influence of the parameters on the wall mass fraction $\omega_{1,w}$ is presented in Fig. 6. The asymptotic value of the mass fraction depends only on the ratio U_2/U_1, ρ_1/ρ_2, and R_1/R for incompressible flow; hence the effect of the other parameters is felt in the initial mixing region only. Faster mixing is associated with a narrower mixing region, hence, an increase of U_2/U_1 or a decrease of ρ_1/ρ_2, that increase mixing, decreases the wall mass fraction $\omega_{1,w}$. A decrease of R_1/R results in a narrower inner jet and hence, the wall mass fraction $\omega_{1,w}$ is lowered. The wall mass fraction $\omega_{1,w}$ decreases as $N_{Re,2}$ or $N_{Sc,2}$ increase, the effect of $N_{Sc,2}$ being more pronounced.

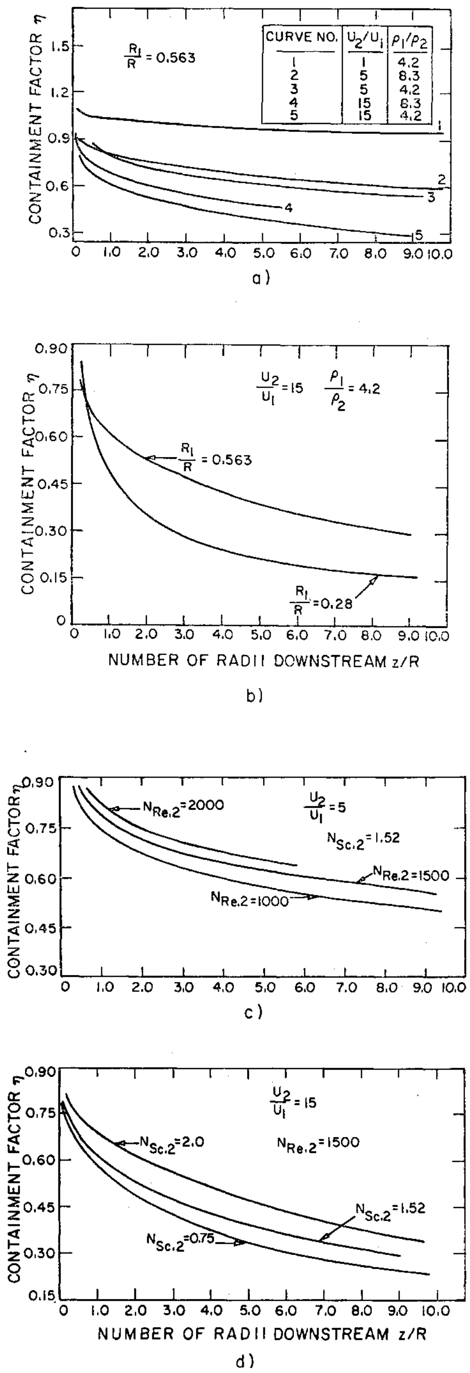

Fig 7 Containment factor vs downstream distance: a) and b) $N_{Re,2} = 1500$, $N_{Sc,2} = 1.52$, $\mu_2/\mu_1 = 1.48$; c) and d) $\rho_1/\rho_2 = 4.2$, $R_1/R = 0.563$, $\mu_2/\mu_1 = 1.48$.

Effects of Flow Parameters on η¶

Figure 7 presents the effects of the parameters on the containment factor η. Eta increases as U_2/U_1 decreases or as ρ_1/ρ_2 and R_1/R increase, indicating a reduction in the depletion of the inner jet. Increase of $N_{Re,2}$ or $N_{Sc,2}$ increases η because higher $N_{Re,2}$ implies lower viscous interaction and higher $N_{Sc,2}$ results in slower diffusion.

¶ η = mass of species 1 in a given volume (between the entrance section and a section downstream) of the confining duct divided by the mass of species 1 in the same volume had there been no mixing. Expressed mathematically

$$\eta = 2\pi \int_0^z \int_0^R \rho\omega_1 r dr dz \Big/ \rho_{p,1}\pi R_1^2 z$$

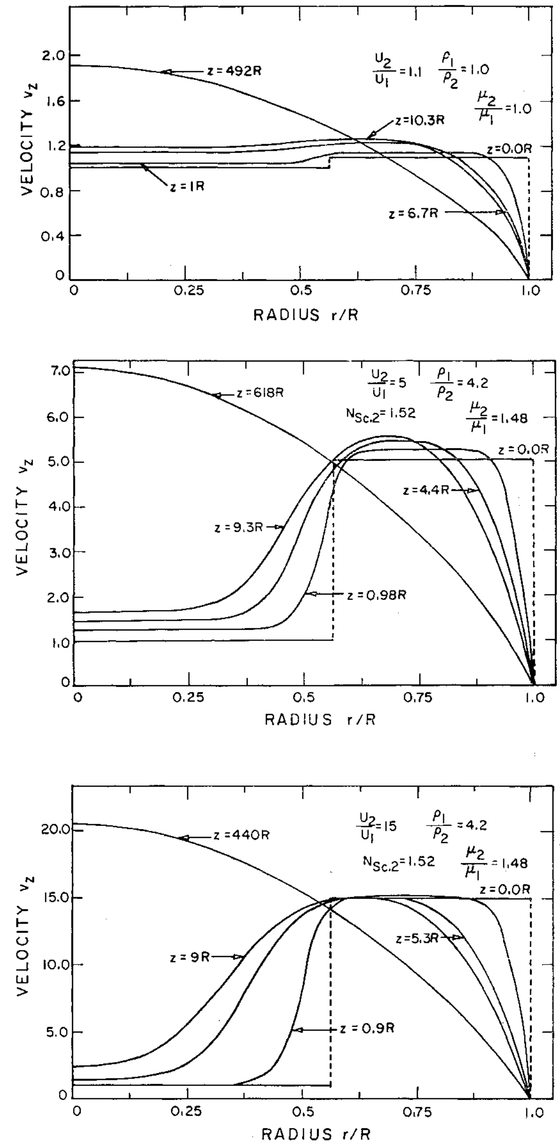

Fig. 8 Axial velocity profiles; $R_1/R = 0.563$, $N_{Re,2} = 1500$.

Velocity and Mass Fraction Fields for Typical Runs

Figure 8 shows the development of the axial velocity profiles; the developing profiles of mass fraction ω_1 are presented in Fig. 9. An increase in developing length results from a decrease in U_2/U_1 or an increase in ρ_1/ρ_2. A decrease in U_2/U_1 leads to more gradual change of the centerline values, i.e., slower mixing and wider jet. An experimental investigation of coaxial turbulent mixing of heterogeneous jets has been recently completed by Zawacki and Weinstein.[15] The measured effects of U_2/U_1 and ρ_1/ρ_2 on the jet mixing agree qualitatively with the prediction of this analysis.

For a few combinations of the flow parameters near the extremes of the viscosity ratio range, a positive pressure gradient or an oscillatory negative pressure gradient was observed in the initial region. For these cases the fully developed values agreed less favorably and it appears that the initial region was most affected. This behavior is not completely understood as yet and further investigation may be necessary. Also, for the cases investigated, the effect of change in μ_2/μ_1 was small, so that no definite trend of this effect could be established. The original aim of the von Mises transformation or the ϕ transformation was to obtain stable solutions for a wide range of flow parameters. Experience with a similar jet mixing problem in the r-z plane revealed that the range in the ψ-z or the ϕ-z plane was not much

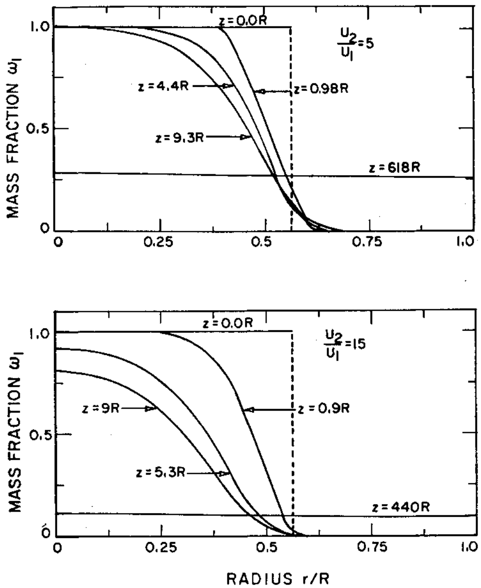

Fig. 9 Mass fraction profiles; ρ_1/ρ_2 = 4.2, R_1/R = 0.563, $N_{Re,2}$ = 1500, $N_{Sc,2}$ = 1.52, μ_2/μ_1 = 1.48.

wider. Therefore, it may be worthwhile to solve the present flow problem in the physical plane where nonuniform entrance profiles can be studied more conveniently.

Conclusions

The results provide detailed information of laminar, incompressible coaxial, confined jet mixing for most of the parameters of practical interest and predict mixing for confined flow configurations that are difficult to investigate experimentally.

An increase in $N_{Re,2}$ or $N_{Sc,2}$ reduces mixing and at the same time, results in decrease of mass fraction at the wall. Also, a decrease in U_2/U_1 or an increase in ρ_1/ρ_2 decreases mixing, while it simultaneously increases the mass fraction at the wall. In the design of gas-core nuclear reactors, minimum mixing as well as low wall mass fraction are desired, hence, a compromise has to be made while choosing the problem parameters. The findings that the length of the mass fraction potential core $L_{\omega 1}$ increases as U_2/U_1 decreases and that the rate of mixing decreases as ρ_1/ρ_2 increases are in qualitative agreement with experimental observations[15] of somewhat similar turbulent flows.

Hence, this study provides trends that may be useful in understanding turbulent jet mixing and may yield information concerning the hydrodynamics of gas-core nuclear reactors. The numerical method has shown sufficient success to warrant further development to study the effects of compressibility and turbulence.

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